Statistical approach for the stochastic evolution of water waves, after level crossing

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Scientific Context

modeling the risk of failure of a marine structure exposed to water wave impacts

Marine structure exposed to wave impacts

2 failure modes

Marine structure exposed to wave impacts

1. risk of exceedance of the material ultimate strength → **single extreme wave impact**
2. risk of failure through fatigue damage → **cumulative effect of a large number of wave impacts**

Marine structure exposed to wave impacts

Modeling the risk: necessary ingredients

marine structure

- geometry of the body
- structural arrangement
- + others (e.g. seakeeping)

sea states

- geographical site
- variables: H_s, T_z, water depth, wave directions

water waves

- physical model or experiments

- number of wave impacts (for a given exposure time)
- kinematics of wave impacts

These two quantities are random

- Slamming loads (hyroelastic coupling)
- Structural stress responses

Risk of failure: two modes

1- Risk of ultimate strength exceedance
2- Risk of fatigue crack initiation / propagation

Marine structure exposed to wave impacts

2 different approaches

1

Model-driven approach
- Linear wave model: Gaussian framework
- 2nd order wave model: non-Gaussianities tractable from the Quadratic Transfer Functions

2

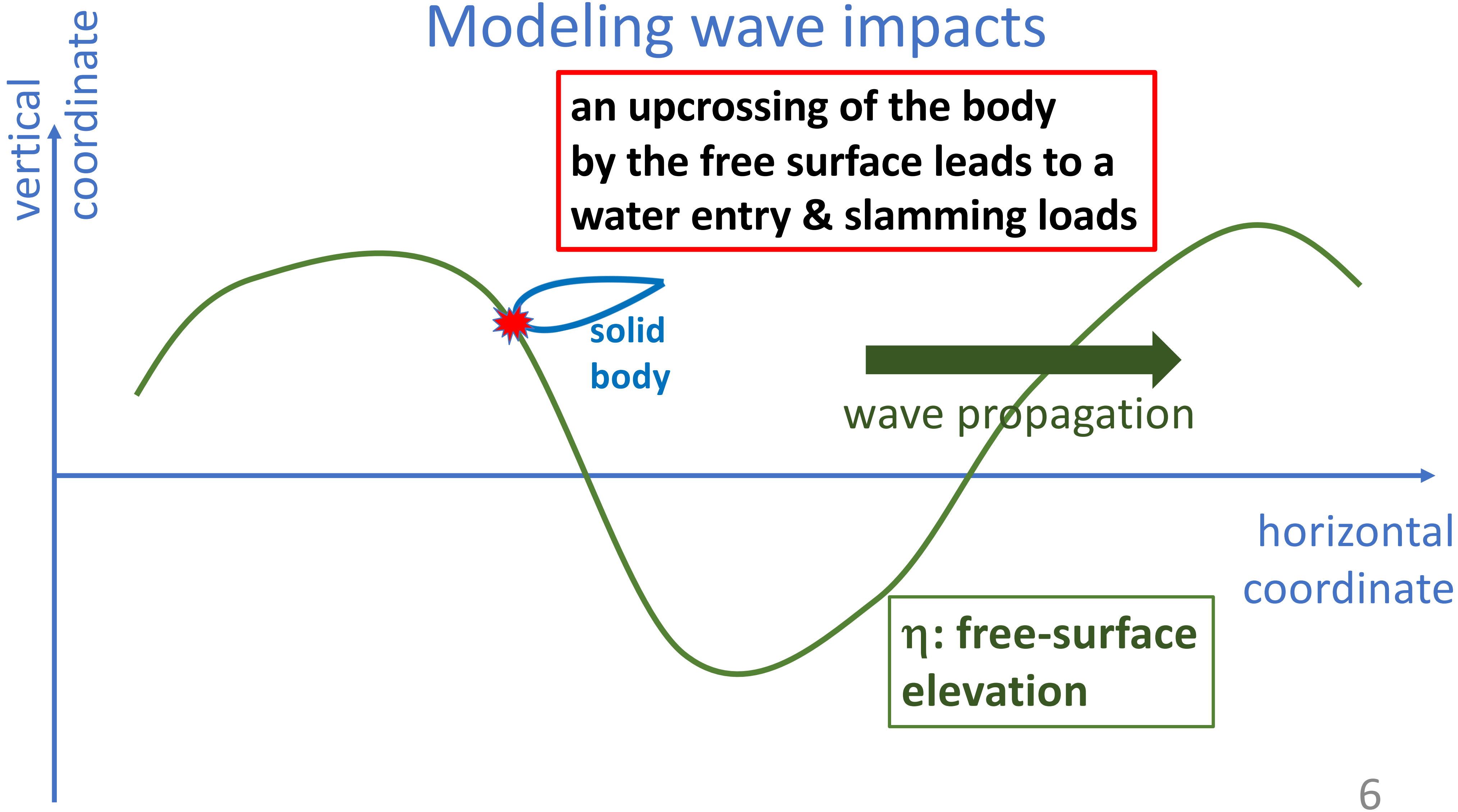
Data-driven approach
- Statistical model fitted to a database
- The database may be from experiments or numerical simulations

Focus of today's talk

water waves
- physical model or experiments

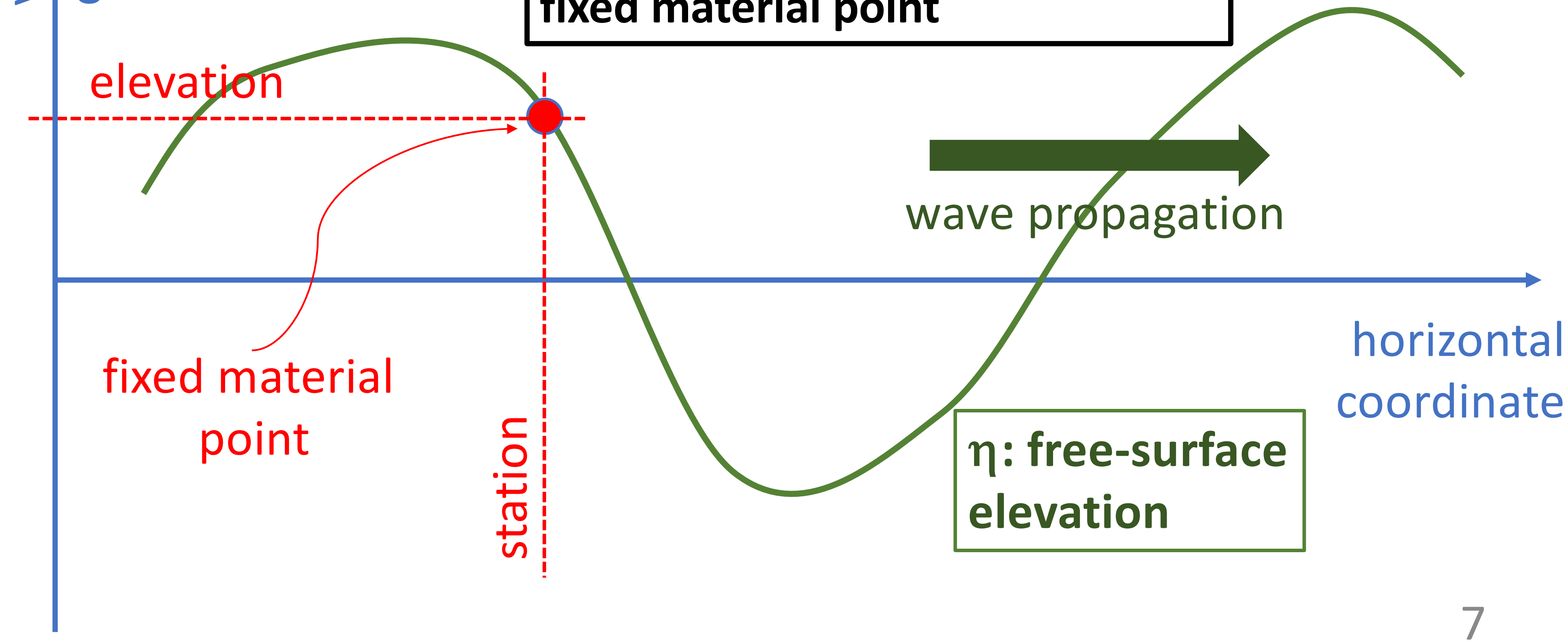
- number of wave impacts (for a given exposure time)
- kinematics of wave impacts

Modeling wave impacts

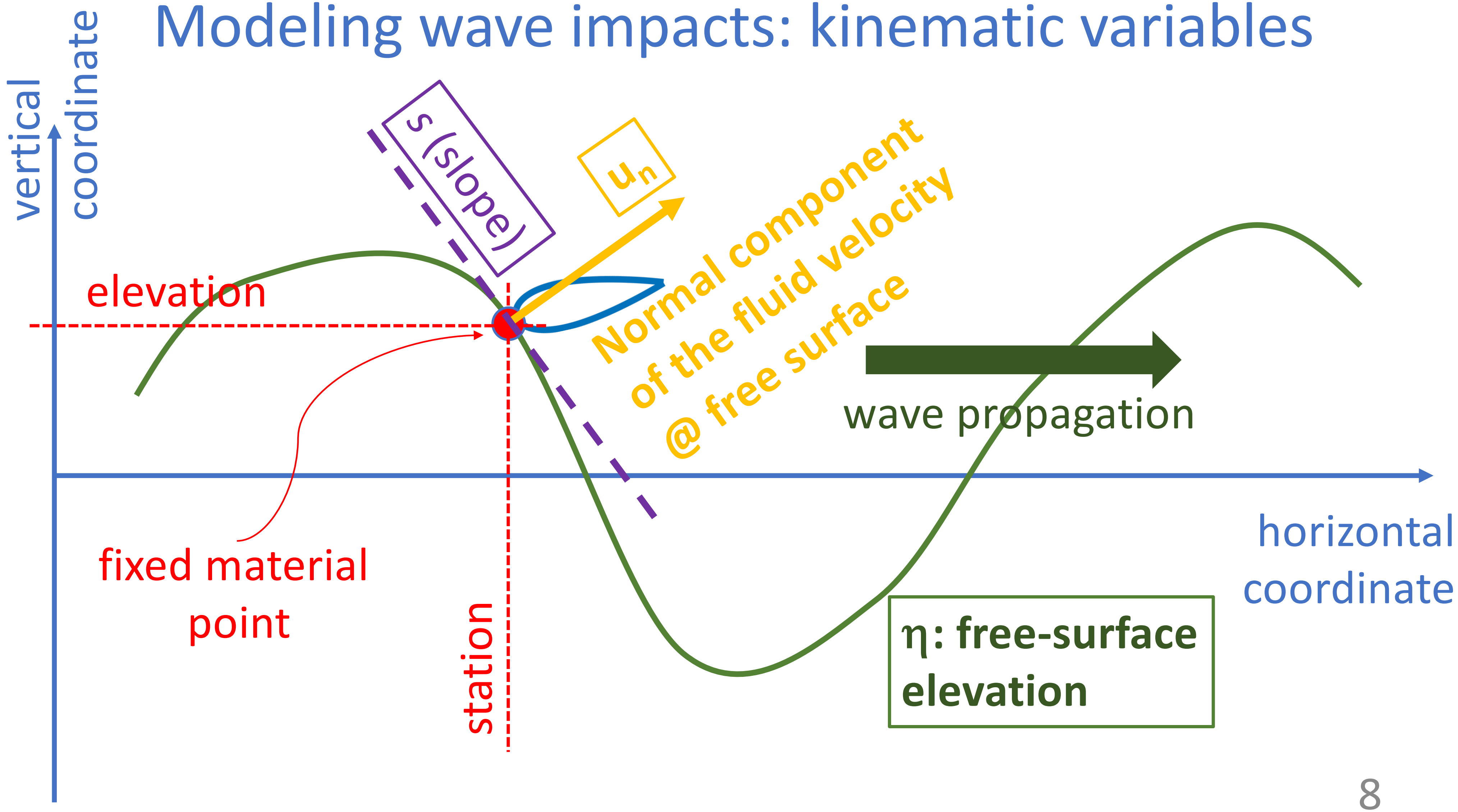


Modeling wave impacts

To model the risk of upcrossing, the solid body is replaced with a fixed material point



Modeling wave impacts: kinematic variables



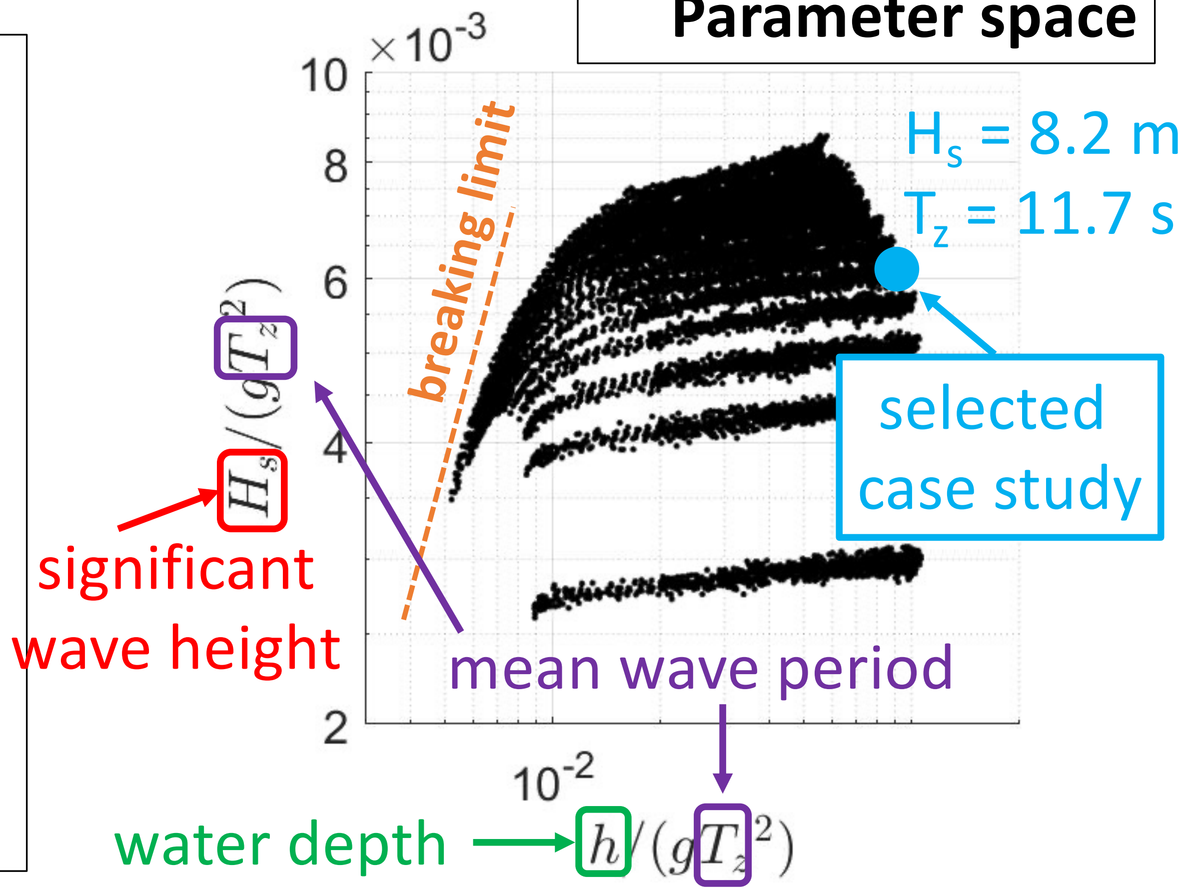
Present work

statistical modelling of water wave evolution, following free-surface level-upcrossing

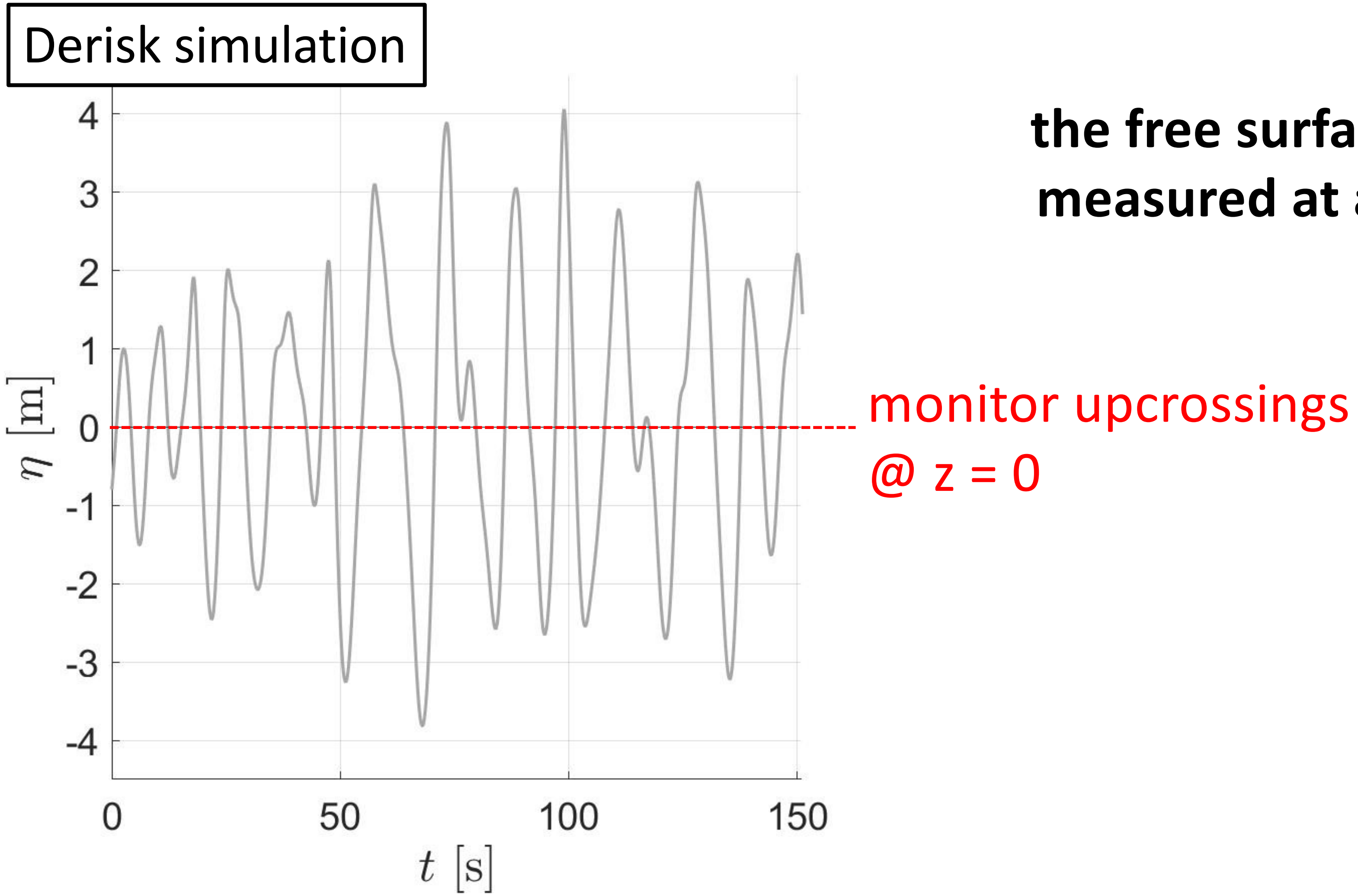
The database: Derisk project (DTU, Pierella et al. 2011)

- Simulations of **irregular seas** with the code OceanWave3D
- **Fully nonlinear** potential flow theory
- **Unidirectional** waves
- Ad hoc breaking wave filter
- Open access on the web
- ≃ **72 hours (physical time)** of simulation for each sea state configuration

Parameter space

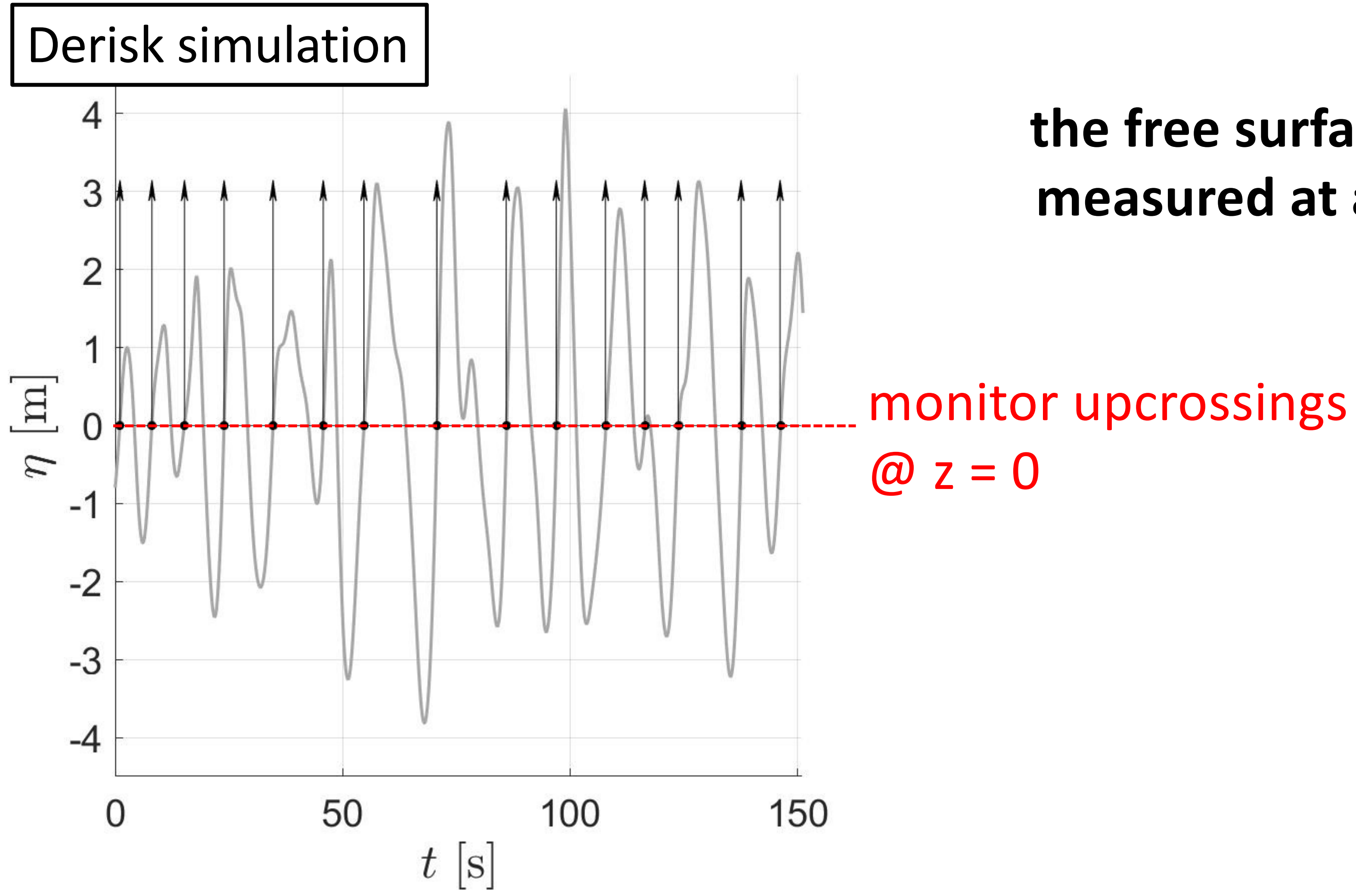


Detect upcrossing events



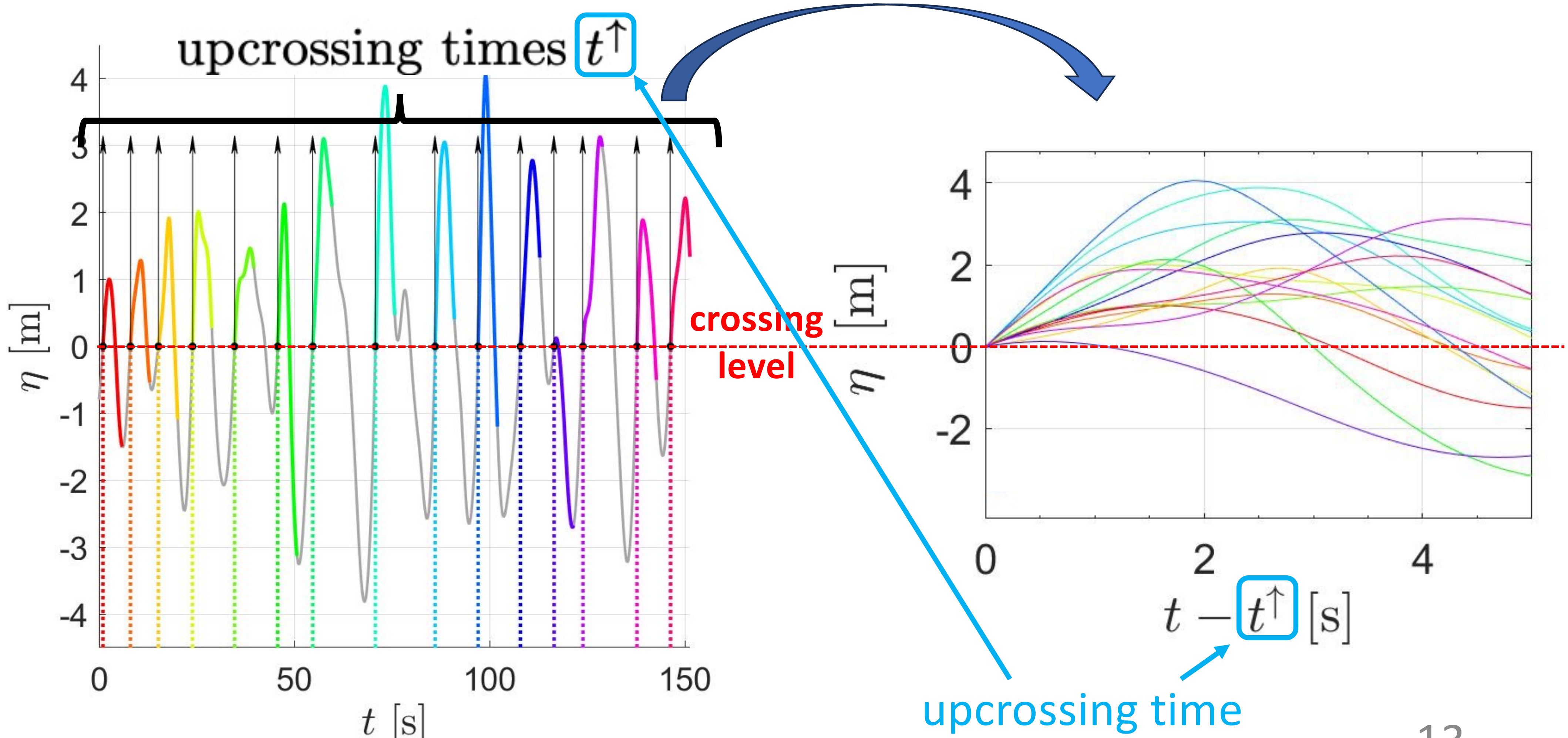
the free surface elevation is measured at a fixed station

Detect upcrossing events



the free surface elevation is measured at a fixed station

Stack the trajectories over a common interval

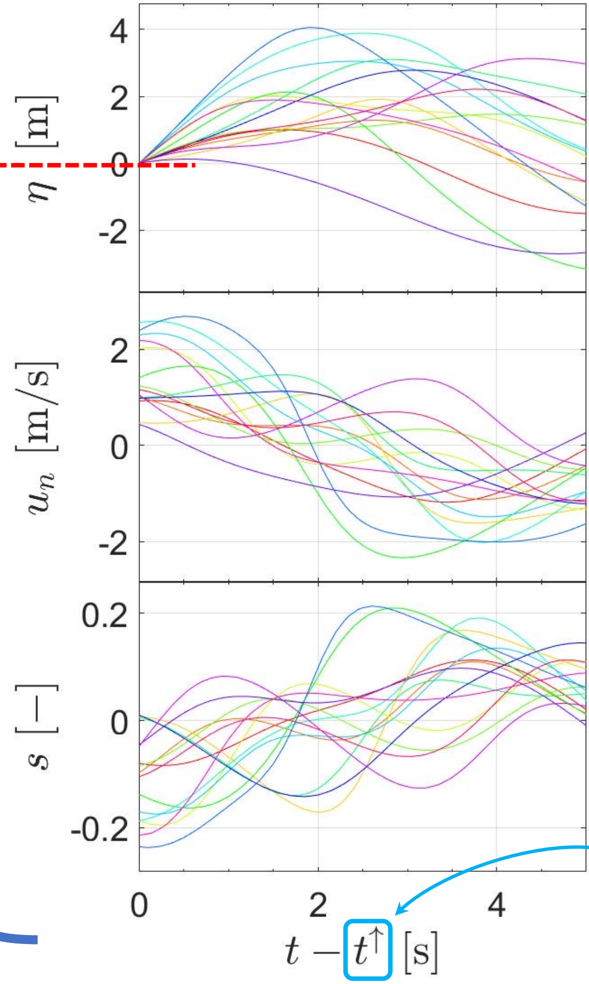


Selected kinematic variables

Multiple realizations of the joint evolution of variables after upcrossing events

Variables are measured at a fixed station

crossing level



➡ η: free surface elevation

➡ u_n: fluid velocity at the free surface (normal component)

➡ s: free surface slope

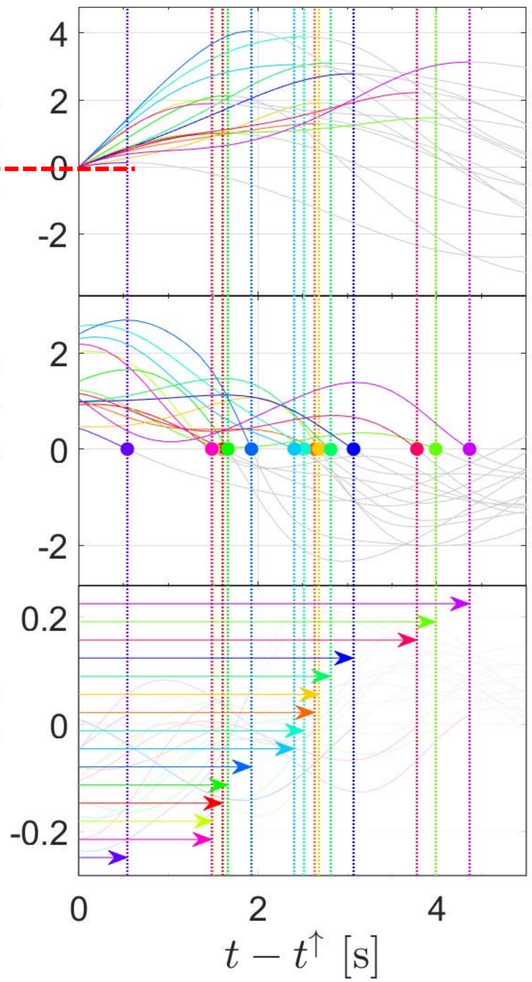
upcrossing time

Define an upcrossing event duration

crossing level

Multiple realizations of the joint evolution of variables after upcrossing events

Variables are measured at a fixed station



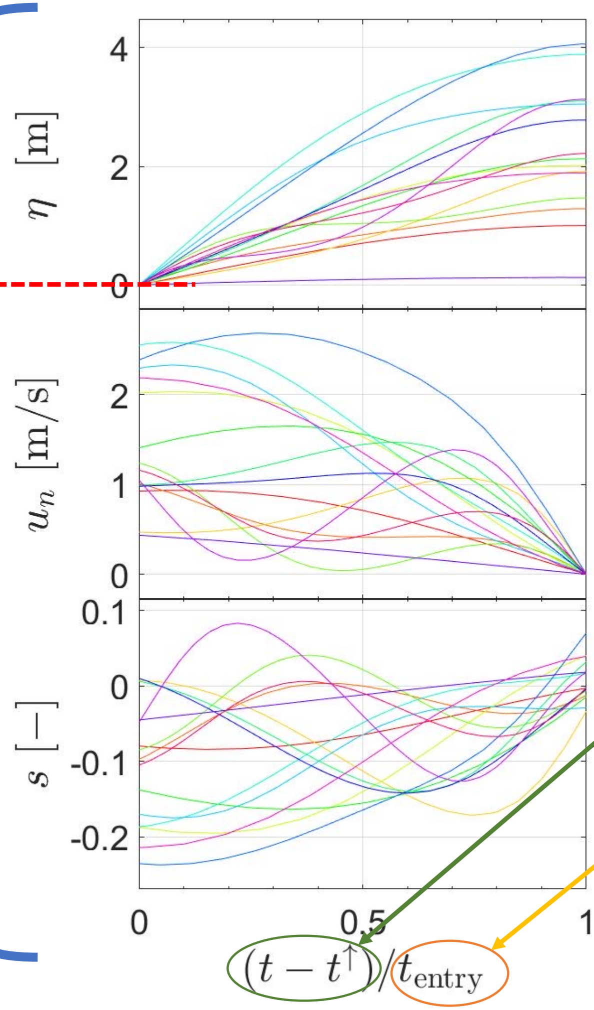
Only the water-entry phase, following upcrossing, is retained → i.e., the post-upcrossing trajectories are retained up to $u_n = 0$ (equivalent to $d\eta/dt=0$)

water entry durations t_{entry}

Rescaling the time parameter

crossing level

Multiple realizations of the joint evolution of variables after upcrossing events



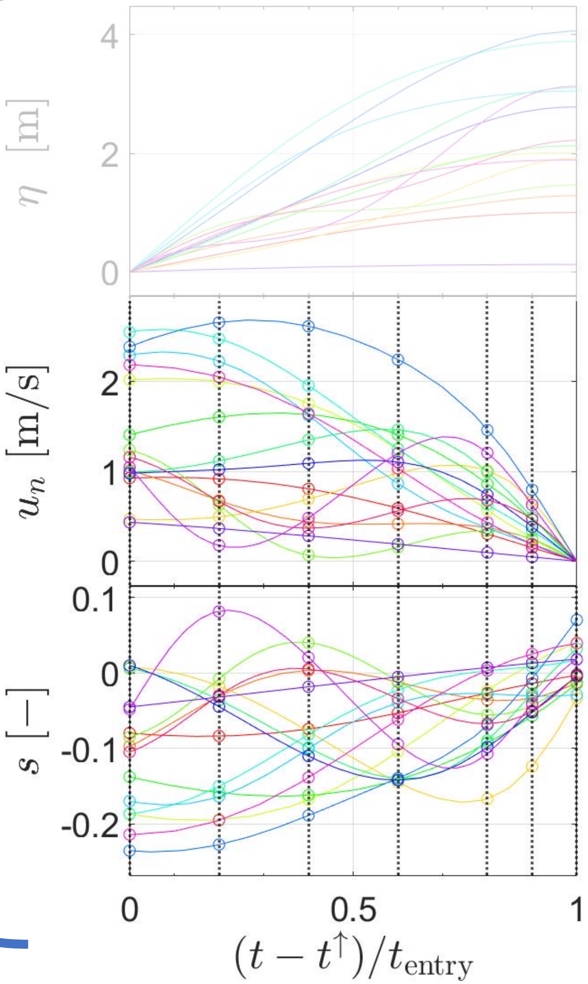
The post-upcrossing times are non-dimensionalized, so that all trajectories have the same support [0,1]

post-upcrossing time

water entry duration

Reduce trajectories to a vector of variables (regressors)

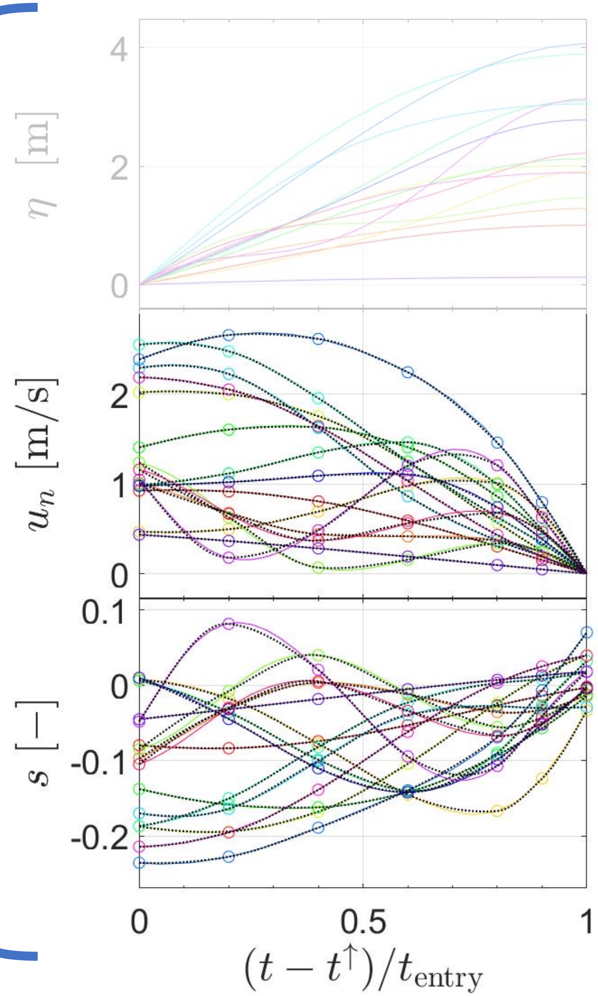
Multiple realizations of the joint evolution of variables after upcrossing events



Choice of regressors which should enable the reconstruction of the stochastic trajectories → values of u_n and s at given nondimensional times

Reduce trajectories to a vector of variables (regressors)

Multiple realizations of the joint evolution of variables after upcrossing events



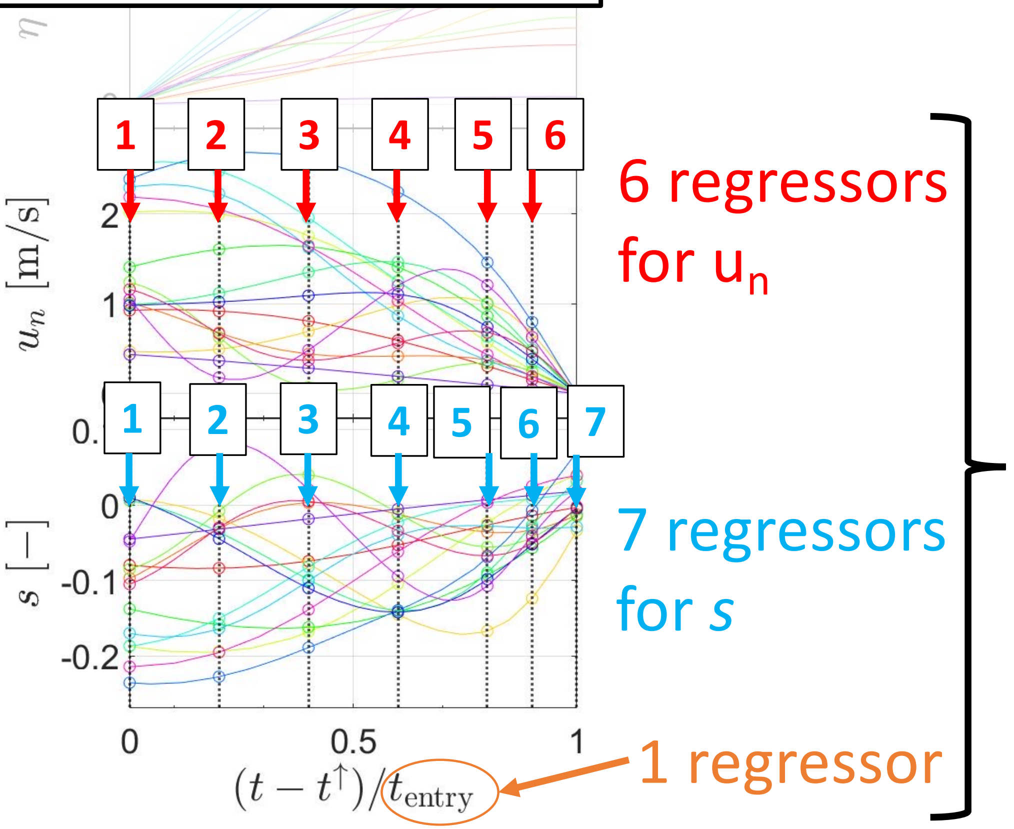
Given a set of regressors, find a method to reconstruct trajectories

→ a **piecewise polynomial interpolation** (modified Akima) works well

Modeling the joint distribution of regressors

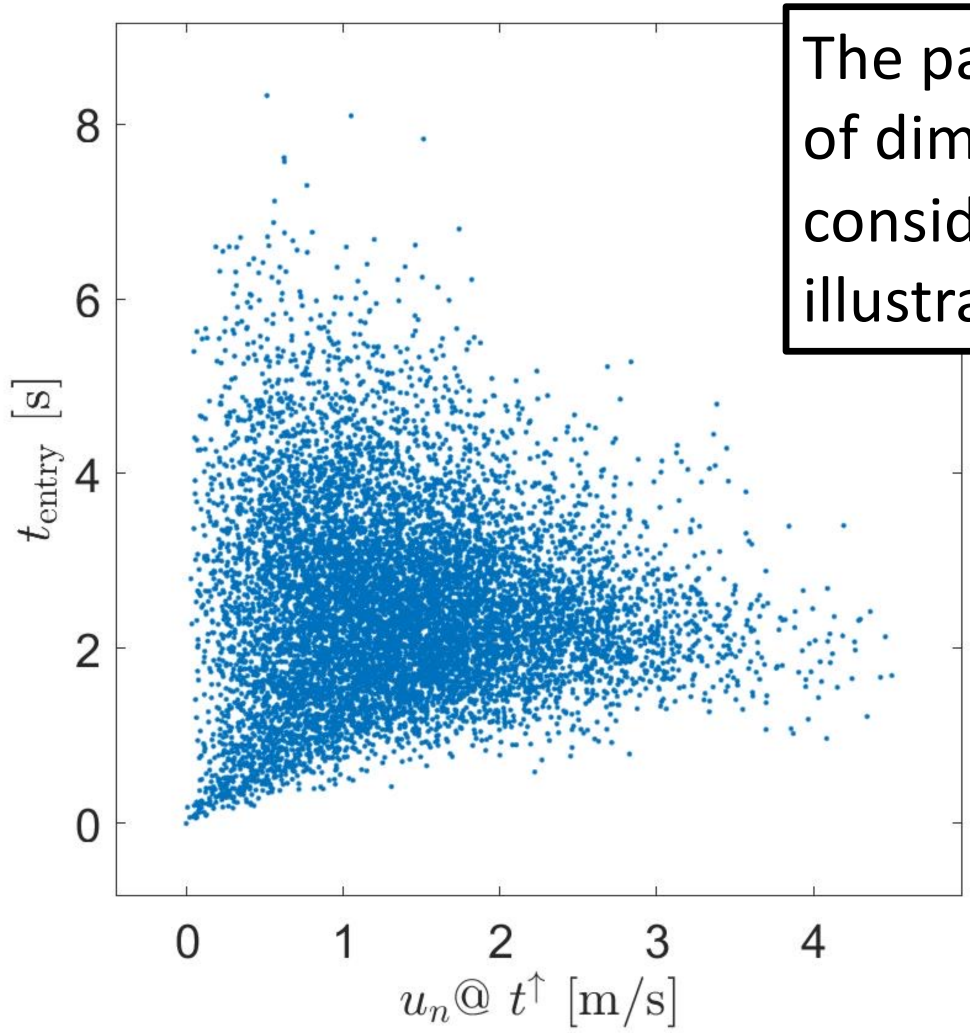
so far so good… now a statistical model is needed for the joint distribution of regressors

- The model should accommodate **high-dimensional spaces**
- **Extreme events** are to be modeled



Joint distribution of 14 variables

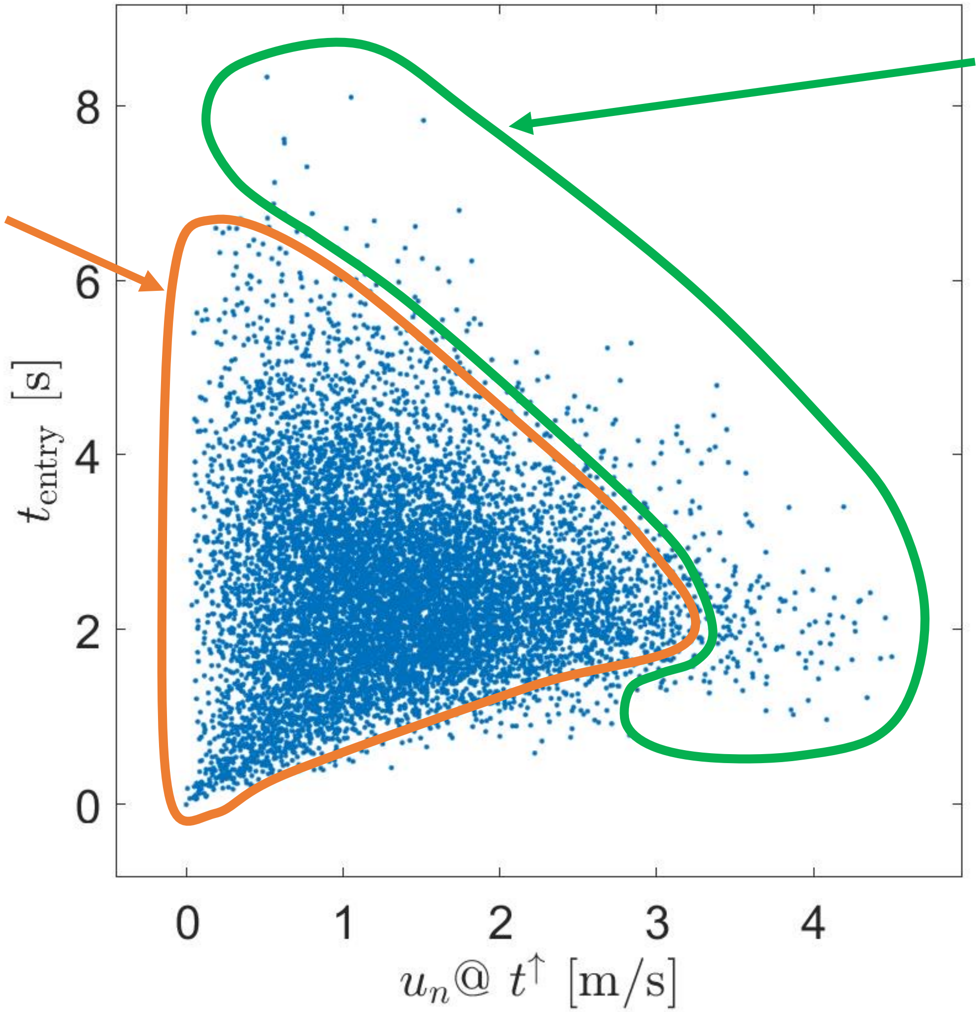
Modeling the joint distribution of regressors



The parameter space if of dimension 14, but let consider a 2D case for illustrative purpose

Modeling the joint distribution of regressors

Bulk of distribution: modeled by a simplified vine copula approach (Thomas Nagler 2014)

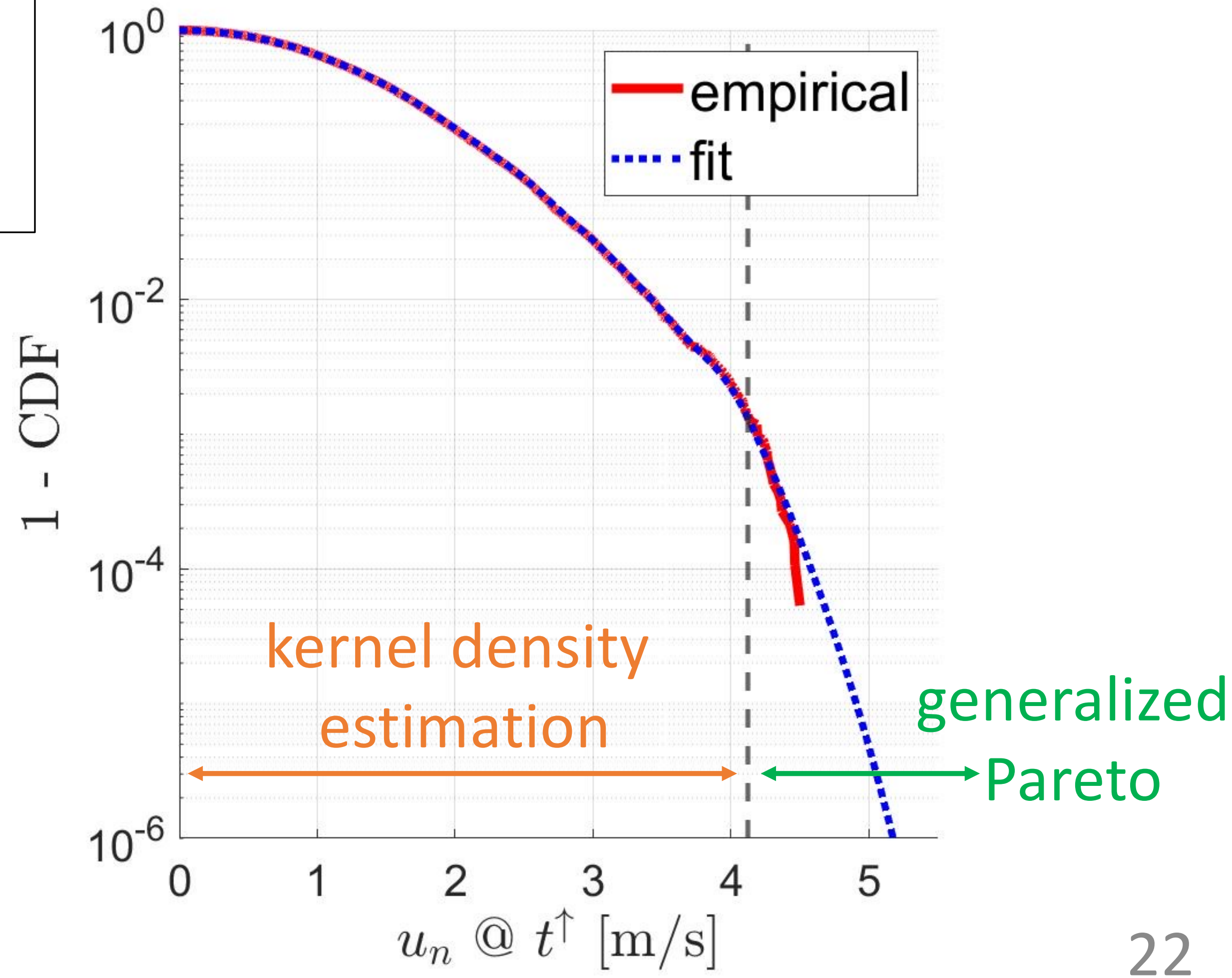


Multivariate tail: modeled by the Heffernan and Tawn (2004) approach

Only the upper tails are modeled

Modeling the joint distribution of regressors

Preliminary step: model the marginal distributions of regressors



Modeling the joint distribution of regressors

Bulk of the distribution: simplified vine copula approach

Preamble: definition of the copula. Example of a 3D distribution

- consider a 3-component random vector (X_1, X_2, X_3)
- marginal distributions are transformed to be uniform
  $(X_1, X_2, X_3) \longrightarrow (U_1, U_2, U_3)$
- the distribution $C(U_1, U_2, U_3)$ defines the copula of (X_1, X_2, X_3)

Modeling the joint distribution of regressors

Bulk of the distribution: simplified vine copula approach

Principle of the vine copula decomposition (Bedford and Cooke 2001)

3D copula density

2D copula densities

$$c(u_1,u_2,u_3) = c_{1,2}(u_1,u_2) \times c_{2,3}(u_2,u_3) \times c_{1,3;2}\big(C_{1|2}(u_1|u_2), C_{3|2}(u_3|u_2); u_2\big)$$

2D copula density
+ u_2 as a parameter
→ ~~trivariate~~ bivariate function

variable 1 variable 2 parameter

➡ Simplifying assumption: a N-dimensional copula density is modeled as the product of N(N-1)/2 bivariate functions (Hobaek Haff et al. 2010)

Modeling the joint distribution of regressors

Bulk of the distribution: simplified vine copula approach

In practice: use of an existing R package

`rvinecopulib` by Thomas Nagler et al.

- Robust
- Parametric models or kernel density estimation for the 2D copula densities

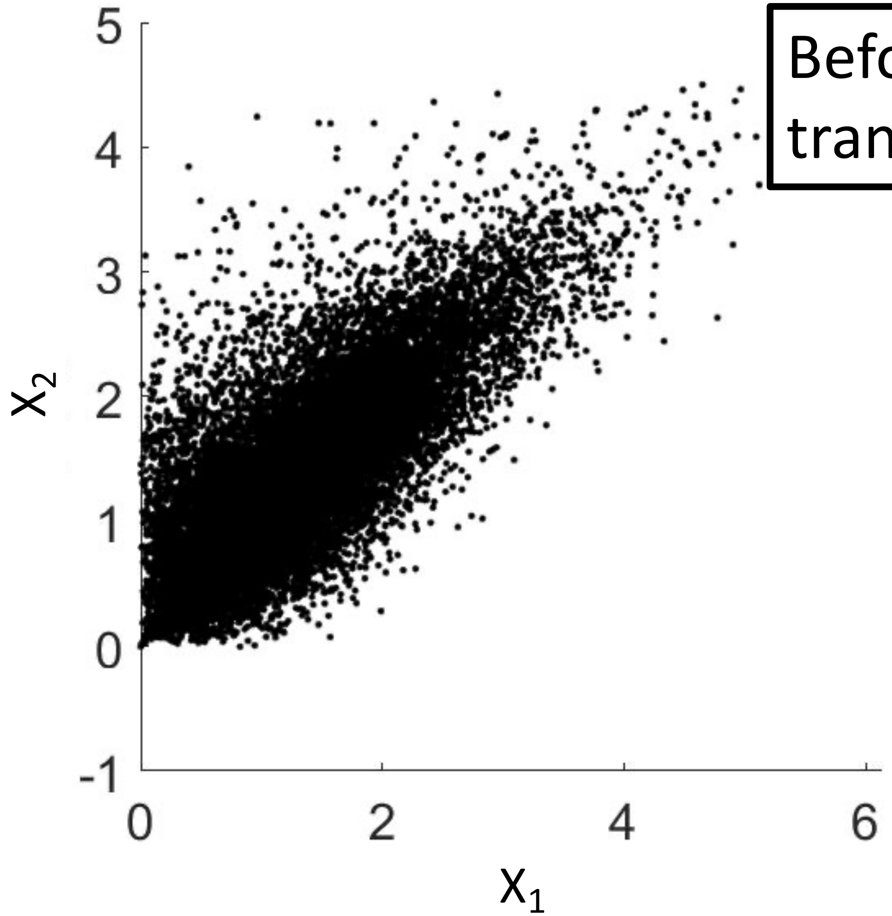
Modeling the joint distribution of regressors

Multivariate tail: Heffernan & Tawn (2004) approach

Example of 2 variables (X_1, X_2), positively correlated

Preliminary step:
transform margins
to Laplace (double
exponential) distribution
(Keef et al. 2013)

$$\begin{cases} X_1 \longrightarrow Y_1 \\ X_2 \longrightarrow Y_2 \end{cases}$$



Before margin transformation

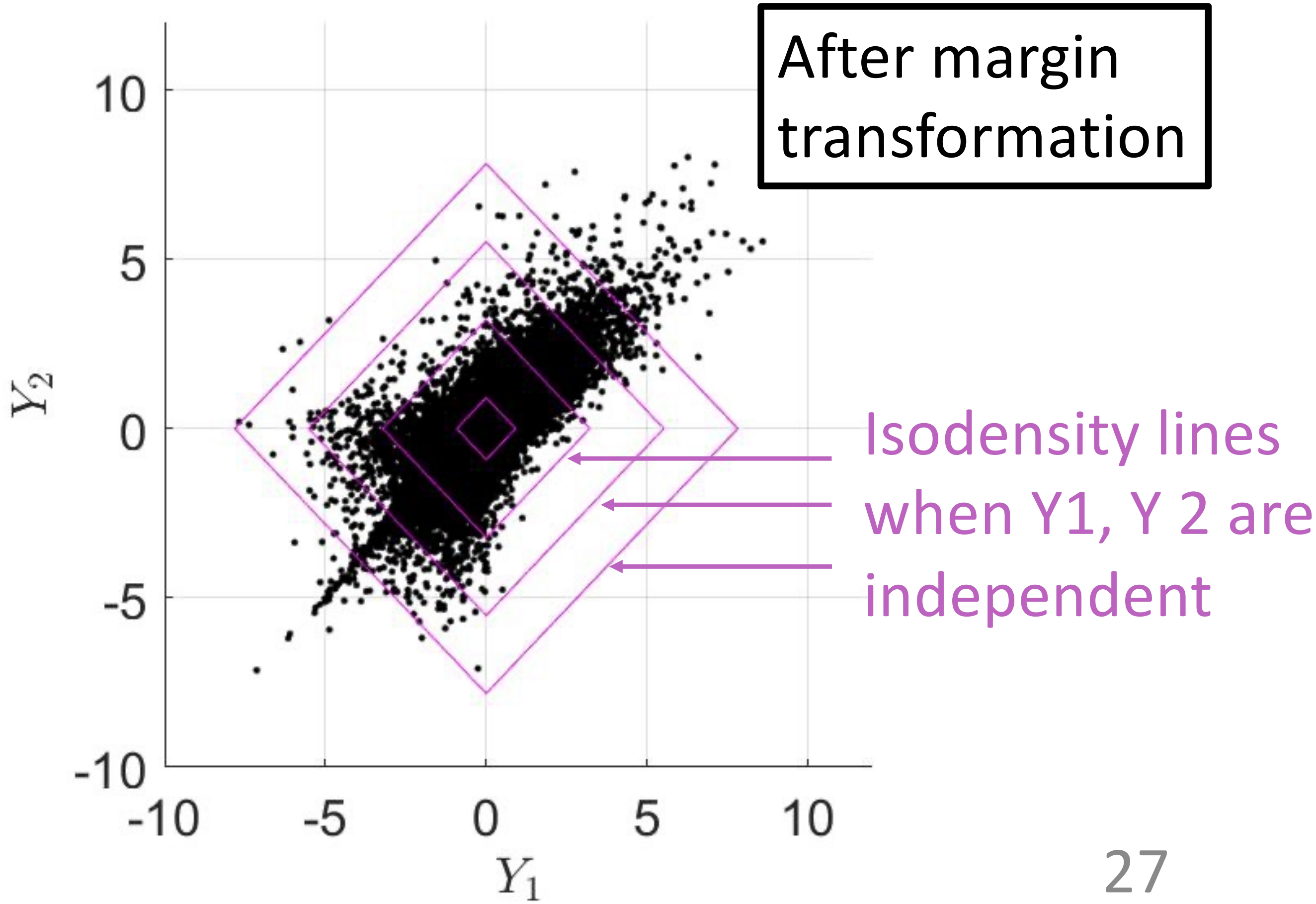
Modeling the joint distribution of regressors

Multivariate tail: Heffernan & Tawn (2004) approach

Example of 2 variables (X_1, X_2), positively correlated

Preliminary step: transform margins to Laplace (double exponential) distribution (Keef et al. 2013)

$X_1 \rightarrow Y_1$
$X_2 \rightarrow Y_2$



Modeling the joint distribution of regressors

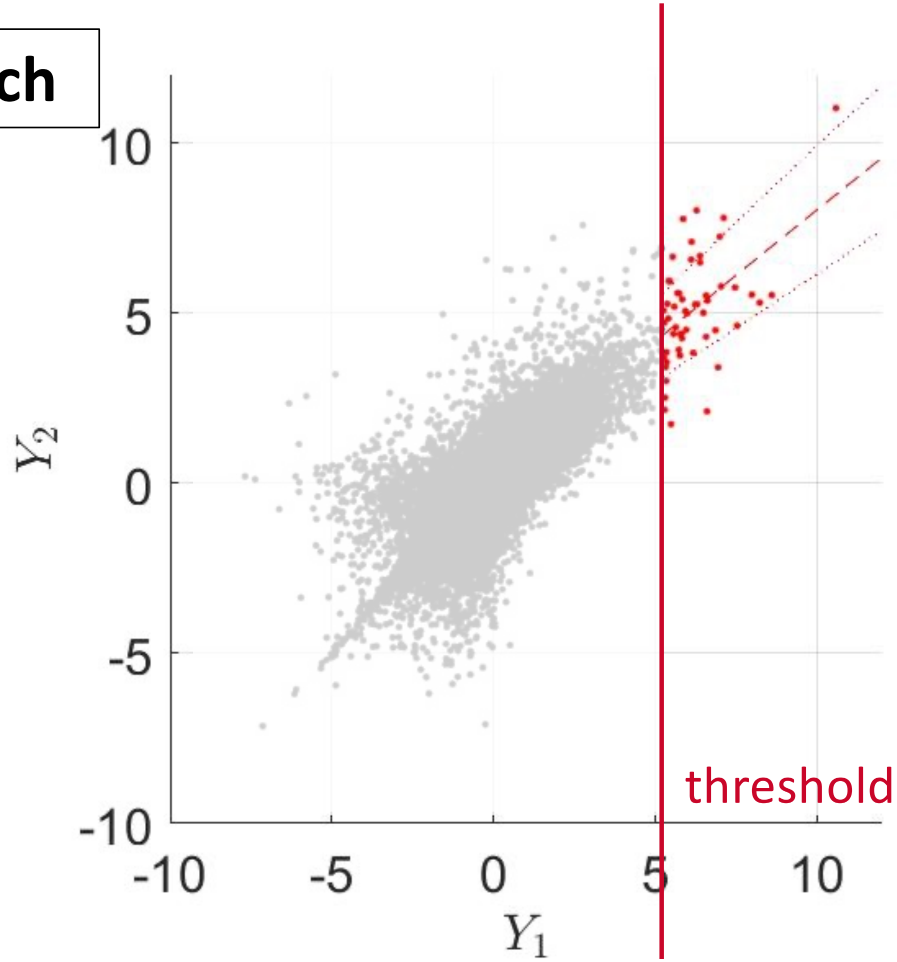
Multivariate tail: Heffernan & Tawn (2004) approach

Principle of the H&T approach

residual — position function — scale function

$$Z_2 = \frac{Y_2 - \mu_{2|1}(Y_1)}{\sigma_{2|1}(Y_1)}$$

with Z_2 independent of Y_1



Modeling the joint distribution of regressors

Multivariate tail: Heffernan & Tawn (2004) approach

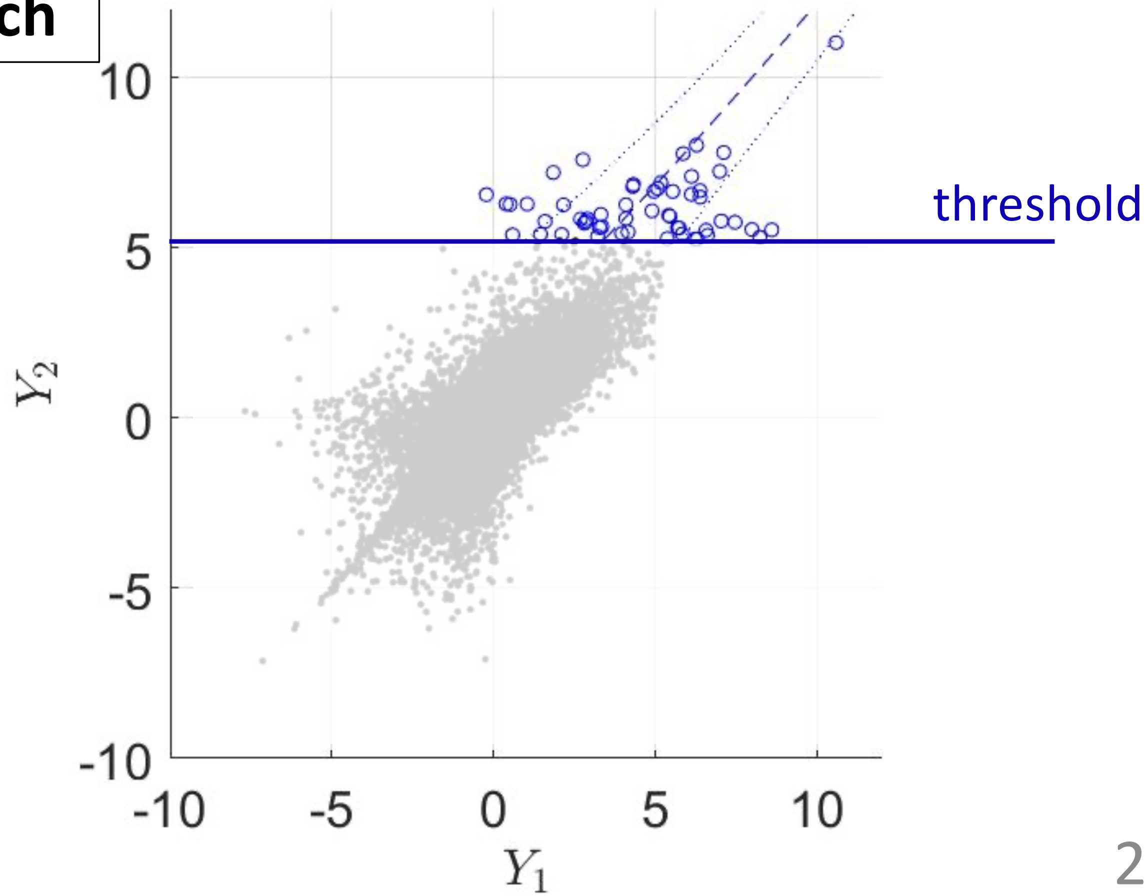
Principle of the H&T approach

residual — position function

$$Z_1 = \frac{Y_1 - \mu_{1|2}(Y_2)}{\sigma_{1|2}(Y_2)}$$

scale function

with Z_1 independent of Y_2



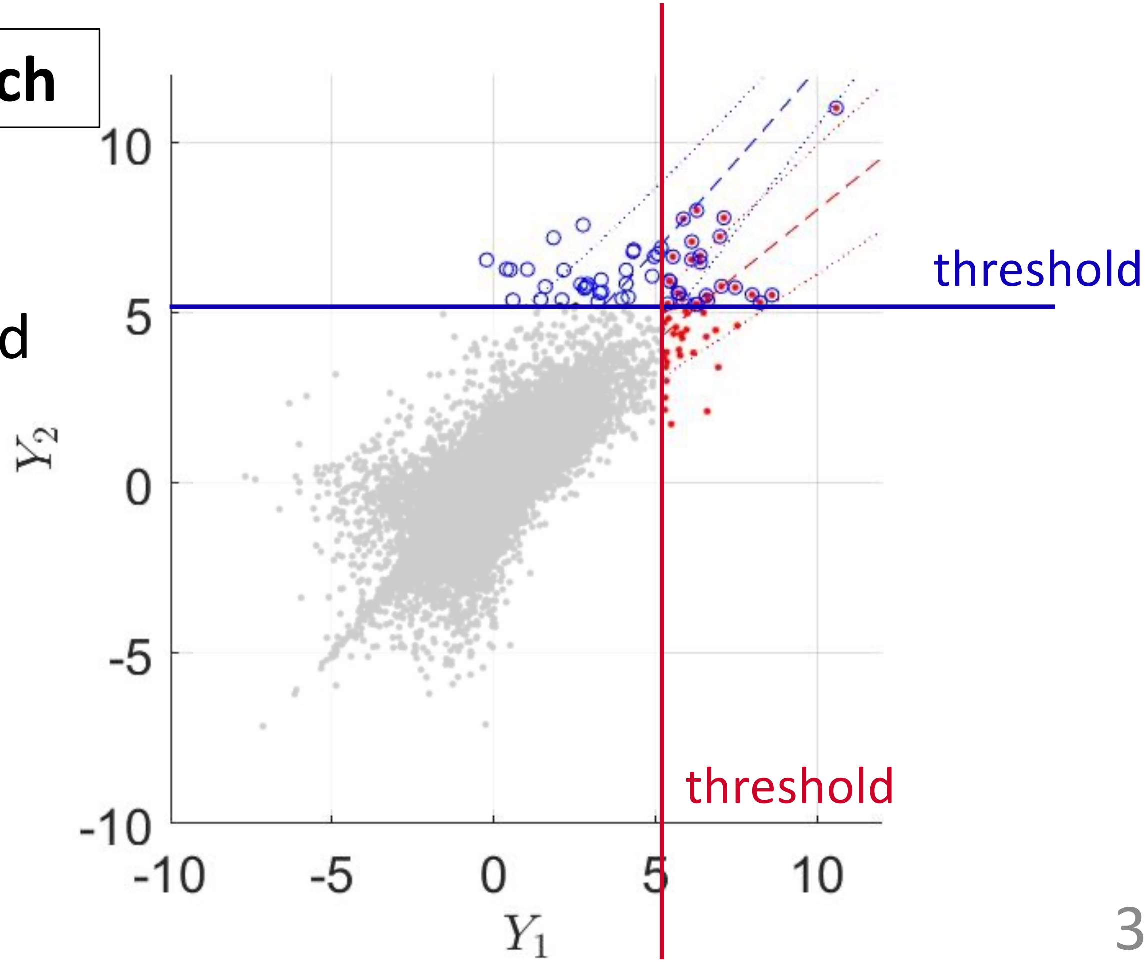
Modeling the joint distribution of regressors

Multivariate tail: Heffernan & Tawn (2004) approach

Principle of the H&T approach

For a pair of variables, two H&T models need to be fitted

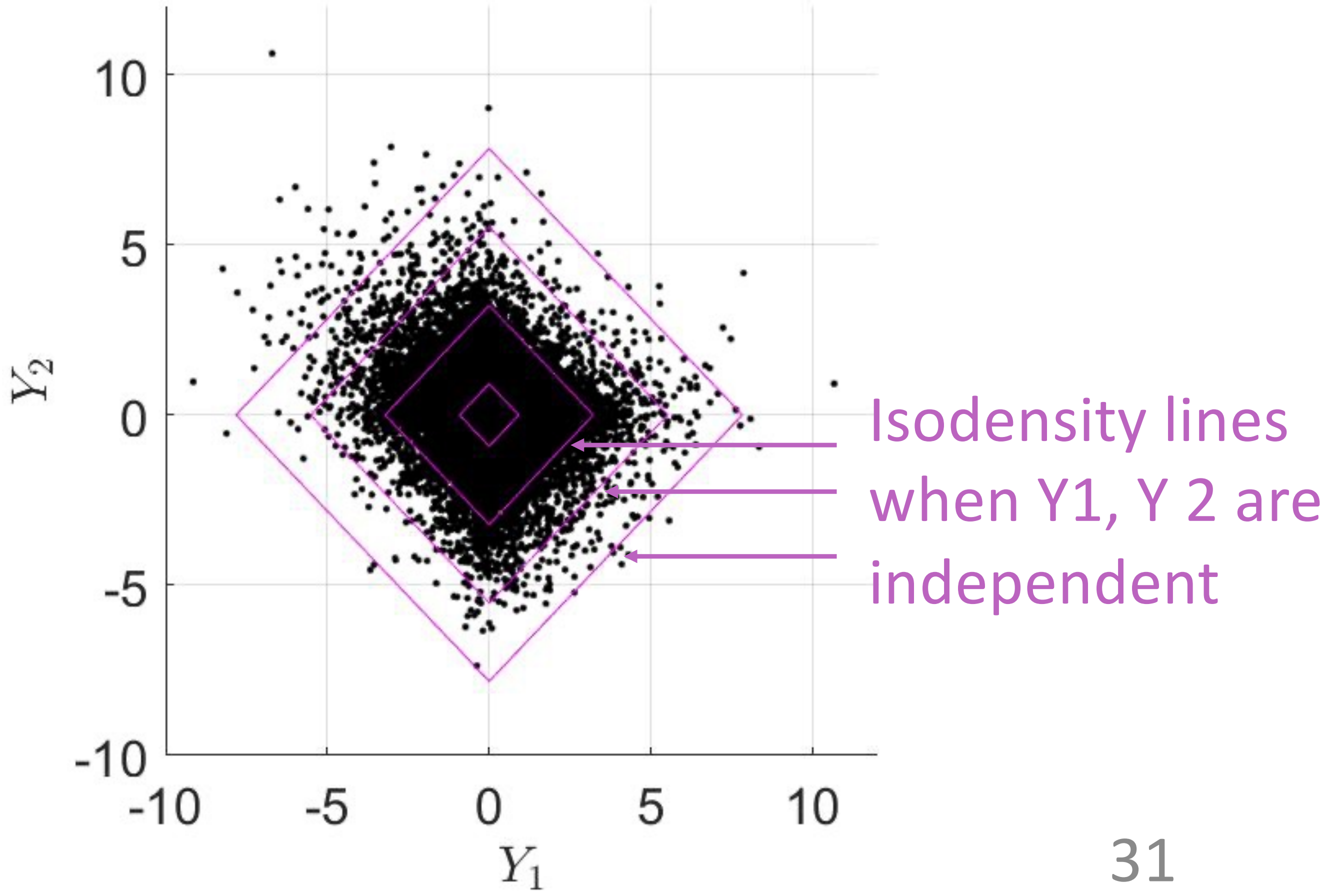
→ For a N-dimensional random vector, N(N-1) H&T models are needed



Modeling the joint distribution of regressors

Multivariate tail: Heffernan & Tawn (2004) approach

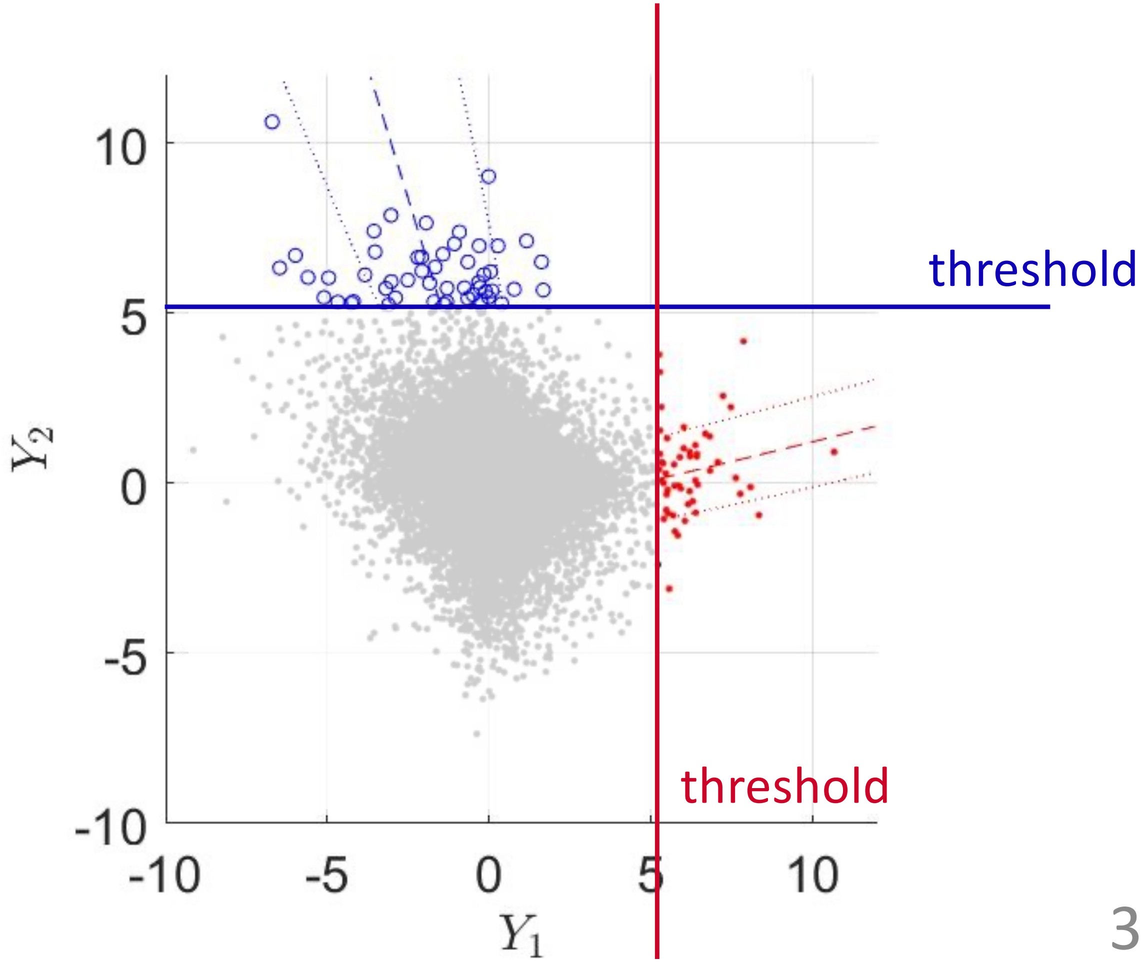
Example of 2 variables (X_1, X_2), negatively correlated



Modeling the joint distribution of regressors

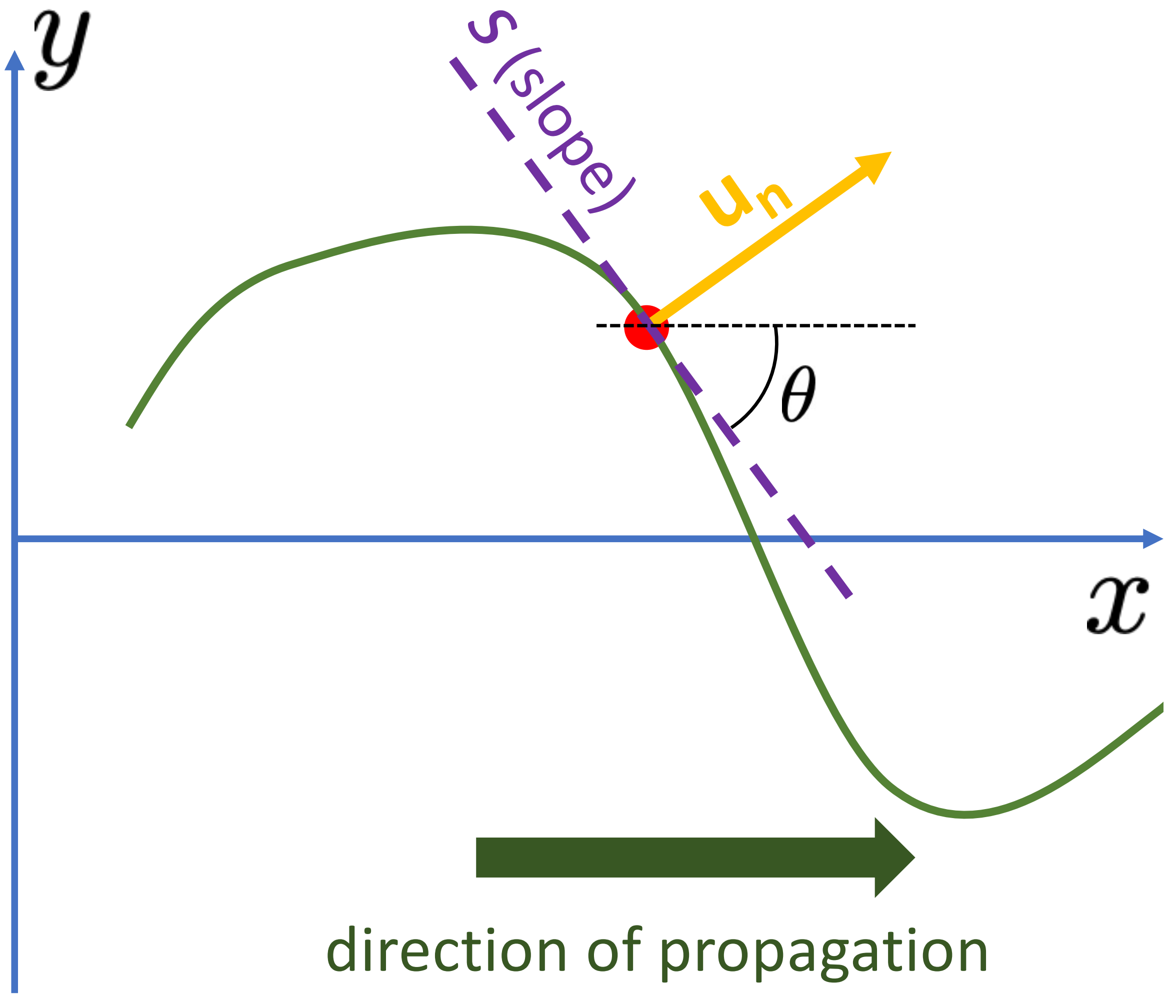
Multivariate tail: Heffernan & Tawn (2004) approach

Example of 2 variables (X_1, X_2), negatively correlated



Results

Testing the model: selection of quantities

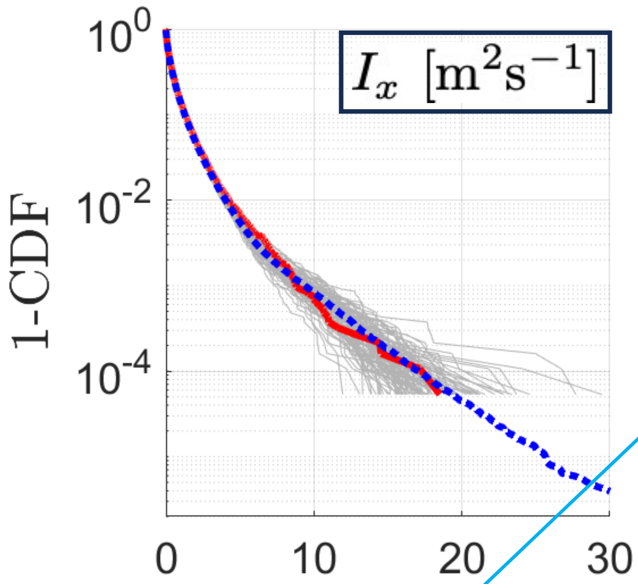


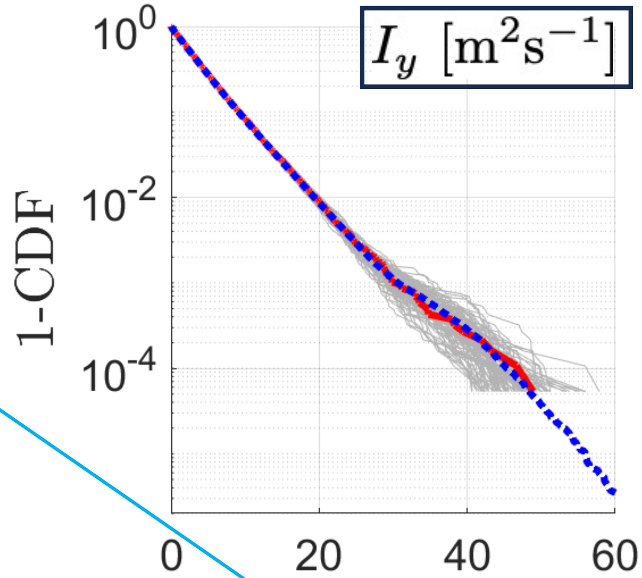
proxies for slamming loads

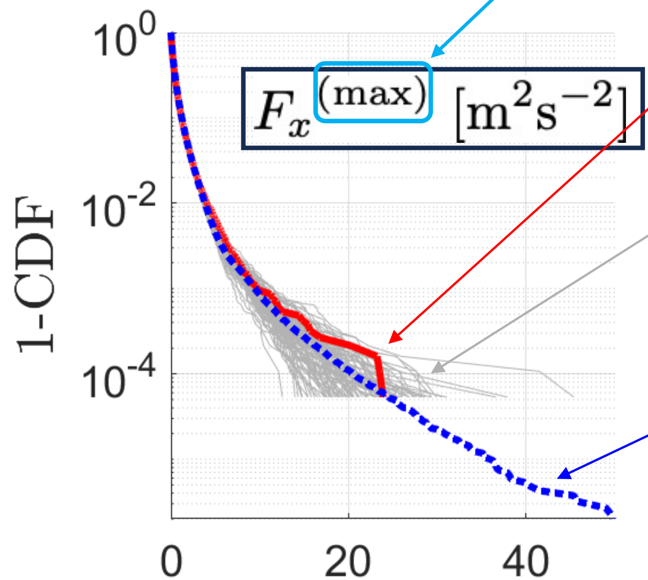
- $F_x = \sin(\theta){u_n}^2$
- $F_y = \cos(\theta){u_n}^2$
- $I_x = \int_0^{t_{\text{entry}}} F_x(t)\mathrm{d}t$
- $I_y = \int_0^{t_{\text{entry}}} F_y(t)\mathrm{d}t$

Preliminary Results

max value during the water entry



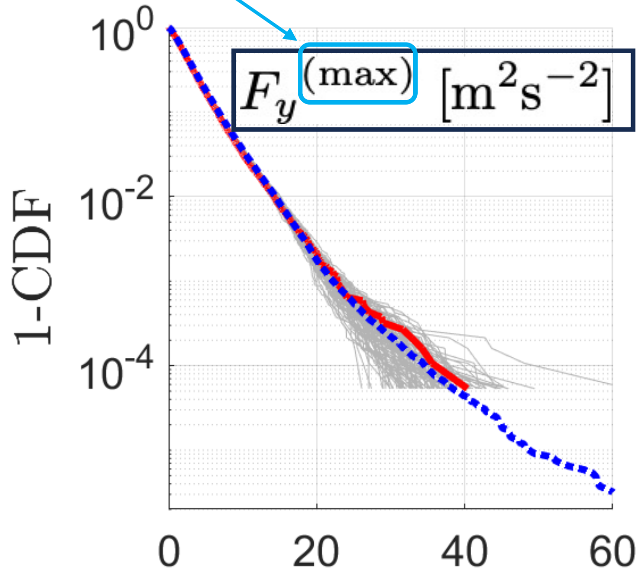




database $N_{obs} = N_0$

statistical model $N_{obs} = N_0$, played 100 times

statistical model $N_{obs} = 100 \times N_0$



Conclusion

- Generic approach
- The model is scalable in terms of dimensionality
- Preliminary results promising

Perspectives

- Test different sea states
- Test different crossing levels